Shipwrecked

The objective of this activity is to determine the relationship between Euler's Method and Riemann Sums approximations for scenarios involving accumlated change.

Euler's Method

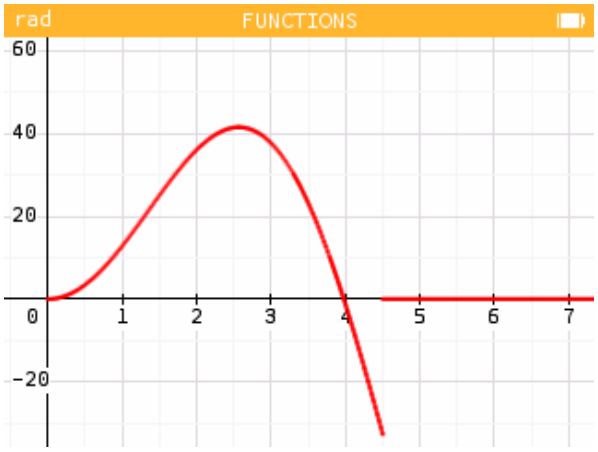
A boat departs from a dock on an island, heading towards open water. The boat speeds up at first, but over time slows down. After about 4 hours, the boat begins to quickly return to the dock but comes to an abrupt stop when it crashes into some rocks. The rate at which the boat travels from the dock over the first 4.5 hours, in miles per hour, is

$$\frac{dB}{dt} = 18t\sin(0.79t)$$

Use Euler's Method with $\Delta t = 0.5$ and $\frac{dB}{dt} = 18t\sin(0.79t)$ to estimate how far the boat is from the dock.

Interpreting the Area Under a Curve

The graph of $\frac{dB}{dt}$ is shown below.



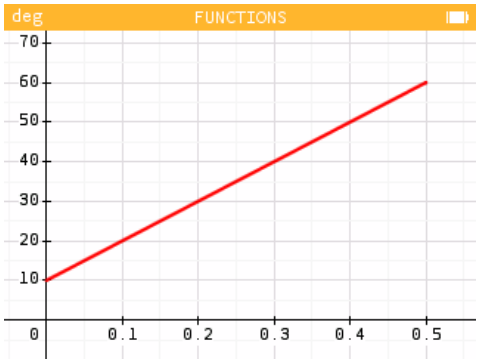
1. Identify the key features of the graph of $\frac{dB}{dt}$. What do these features represent in the context of this scenario?
2. On the graph, sketch left-side rectangles with $\Delta t = 0.5$. Consider the units of the rectangles. What does the area under the curve represent in the context of this problem?

3. Determine the area of each rectangle and sum the areas to approximate the area under the curve of $\frac{dB}{dt}$ on the interval [0,4.5]. Compare your answer to your Euler's Method work.

4. What was the total distance traveled by the boat over the time interval $0 \leq t \leq 4.5$?

Using the Particular Solution

From the same dock on the island, a jet ski heads towards open waters. Initially, it travels at a rate of 10 miles per hour. However, the jet ski malfunctions and continues to accelerate. Thirty minutes later, it is traveling at a rate of 60 miles per hour and needs to be rescued by the Coast Guard. The Coast Guard is stationed on the mainland 20 miles from the island. The speed of the jet ski increases linearly and can be modeled by the differential equation



$$\frac{dJ}{dt} = 100t + 10$$

1. Use left-side rectangles with $\Delta t = 0.1$ hours to estimate how far the jet ski is from the **dock** after 30 minutes.

2. Let J(t) represent the distance that the jet ski is from the mainland. Note that at t = 0, J(t) = 20. Solve the differential equation $\frac{dJ}{dt} = 100t + 10$ by inspection and determine the particular solution appropriate to the given conditions.

3. Use the equation you found to evaluate J(0.5) and J(0). Then calculate J(0.5) - J(0). What does this value represent in the context of this scenario?

4. Compare your answers from questions 1 and 3. How are they related?