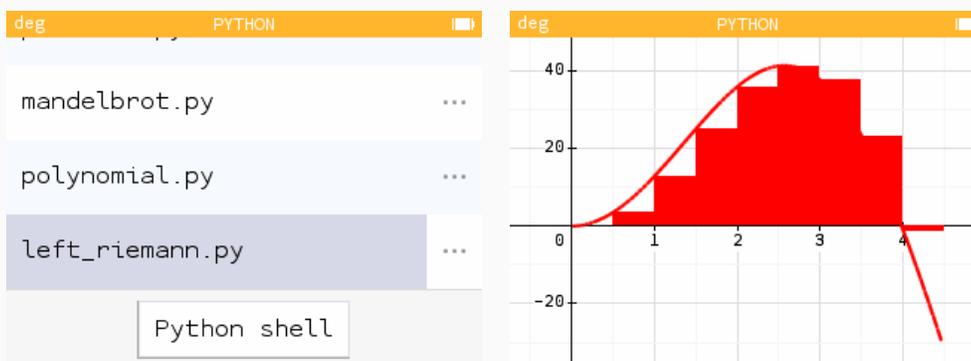


1. Identify the key features of the graph of $\frac{dB}{dt}$. What do these features represent in the context of this scenario?

Location	Description	Context
t = 0 to around t = 2.5	The graph is positive and increasing	The boat is moving away from the dock and accelerating.
Around t = 2.5 to around t = 4	The graph is positive but decreasing	The boat is still moving away from the dock but is slowing down.
Around t = 4 to t = 4.5	The graph is negative and decreasing	The boat is heading back towards the dock and speeding up.
t = 4.5 and on	The graph has a horizontal line of y = 0	The boat is stopped. It hit the rocks.

2. On the graph, sketch left-side rectangles with $\Delta t = 0.5$. Consider the units of the rectangles. What does the area under the curve represent in the context of this problem?

Left-side rectangles can be drawn by hand or using technology. A script can be implemented in Python app.



A python script^a is available online to load onto your calculator.

The units of the width of each rectangle is Δt . The units of the height of each rectangle is $\frac{\Delta B}{\Delta t}$. Thus the units for the area are $\frac{\Delta B}{\Delta t} * \Delta t = \Delta B$. In other words, the area of the rectangles represent the change in the boat's distance from the dock.

^ahttps://my.numworks.com/python/nick-koberstein/left_riemann

3. Determine the area of each rectangle and sum the areas to approximate the area under the curve of $\frac{dB}{dt}$ on the interval $[0,4.5]$. Compare your answer to your Euler's Method work.

Rectangle	Width	Height	Area
1	0.5	0	0
2	0.5	3.463	1.732
3	0.5	12.786	6.393
4	0.5	25.015	12.508
5	0.5	35.998	18
6	0.5	41.374	20.687
7	0.5	37.653	18.827
8	0.5	23.169	11.585
9	0.5	-1.325	-0.663

rad CALCULATION

$$\sum_{i=0}^8 \left(0.5 \times 18 \times \left(\frac{i}{2} \right) \times \sin \left(0.79 \times \left(\frac{i}{2} \right) \right) \right)$$

89.0667727

The change in the boat's distance from the dock is 89.067 miles. That is, the boat is 89.067 miles away from the dock. This is the same value obtained from Euler's Method with $\Delta t = 0.5$.

4. What was the total distance traveled by the boat over the time interval $0 \leq t \leq 4.5$?

To determine the total distance traveled, we consider the absolute value of the height of each rectangle.

rad CALCULATION

$$\sum_{i=0}^8 \left(0.5 \times \left| 18 \left(\frac{i}{2} \right) \sin \left(0.79 \times \left(\frac{i}{2} \right) \right) \right| \right)$$

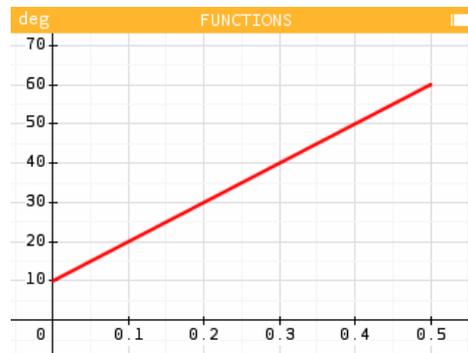
90.3920268

The boat traveled a total distance of 90.392 miles before hitting the rocks.

Using the Particular Solution

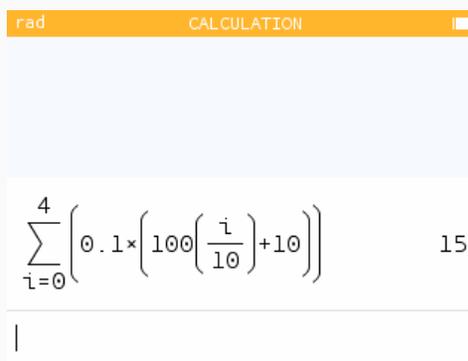
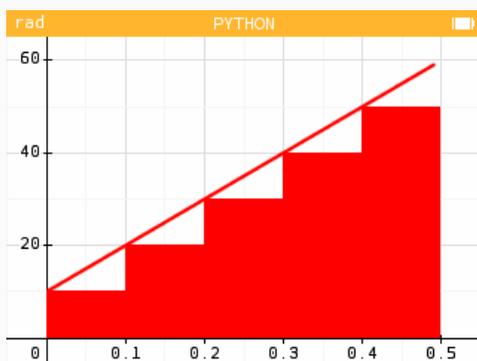
From the same dock on the island, a jet ski heads towards open waters. Initially, it travels at a rate of 10 miles per hour. However, the jet ski malfunctions and continues to accelerate. Thirty minutes later, it is traveling at a rate of 60 miles per hour and needs to be rescued by the Coast Guard. The Coast Guard is stationed on the mainland 20 miles from the island. The speed of the jet ski increases linearly and can be modeled by the differential equation

$$\frac{dJ}{dt} = 100t + 10$$



1. Use left-side rectangles with $\Delta t = 0.1$ hours to estimate how far the jet ski is from the dock after 30 minutes.

Left-side rectangles can be drawn by hand or using technology. A script can be implemented in Python app.



A python script^a is available online to load onto your calculator.

The jet ski is approximately 15 miles away from the dock after 30 minutes.

^ahttps://my.numworks.com/python/nick-koberstein/jet_ski

2. Let $J(t)$ represent the distance that the jet ski is from the mainland. Note that at $t = 0$, $J(t) = 20$. Solve the differential equation $\frac{dJ}{dt} = 100t + 10$ by inspection and determine the particular solution appropriate to the given conditions.

The general solutions for $\frac{dJ}{dt} = 100t + 10$ is $J(t) = 50t^2 + 10t + c$. Using the initial conditions (0,20)

$$20 = 50(0)^2 + 10(0) + c$$

$$20 = c$$

We have the particular solution $J(t) = 50t^2 + 10t + 20$.

3. Use the equation you found to evaluate $J(0.5)$ and $J(0)$. Then calculate $J(0.5) - J(0)$. What does this value represent in the context of this scenario?

$$J(0.5) = 37.5$$

$$J(0) = 20$$

$$J(0.5) - J(0) = 37.5 - 20 = 17.5$$

$J(0)$ represents the distance that the jet ski is from the mainland at time 0. This is when the jet ski is at the dock. $J(0.5)$ represents the distance that the jet ski is from the mainland at time 0.5. The difference between these two values represents how far the jet ski has traveled from the dock over the time interval.

4. Compare your answers from questions 1 and 3. How are they related?

The answers are similar. In question 1, we are estimated the change in distance using left-side rectangles. However, because the graph of the differential equation is increasing, the left-side rectangles are an underestimate. In question 3, we have the solution function. We can find the actual values of $J(t)$ and the difference between two values. Thus question 3 is giving the actual distance that the jet ski is from the dock at the end of the time interval.