Fair Games

The objective of this activity is to compute probabilities using probability distributions of discrete random variables and to calculate and interpret the mean (expected value) and standard deviation of discrete random variables.

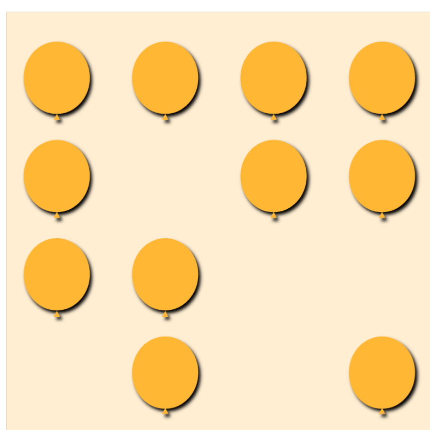
State Fairs are one of the most well known fall events with their funnel cakes, exhilarating rides, music events, and games! Fair games often entice guests with their promise of oversized stuffed animals, hats, jerseys, and other exciting prizes. However, not all fair games are actually ”fair”!

Many games of chance are designed so that the game attendant has a better chance of winning that the players. Every now and then, the players will win, which gives them confidence to carry on playing! However, if the game is weighted towards the game attendant, then they will always win in the long run!

A **fair game** is a game in which there is an equal chance of winning or losing. We can say that a game is fair if the probability of winning is equal to the probability of losing.

Balloon Pop

Consider a game where balloons are attached to a board and the player throws a dart in hopes of popping a balloon.



1. If the dimensions of the board are 3.5 feet wide and 3.5 feet high, and the diameter of each balloon is 8 inches, what is the probability that a randomly thrown dart hits a balloon?

2. Assume it costs \$1 to play the game and that if the player hits a balloon, they win \$3. Is this a fair game? If not, who has the advantage?

3. Does winning \$4 for hitting the balloon change who has the advantage?

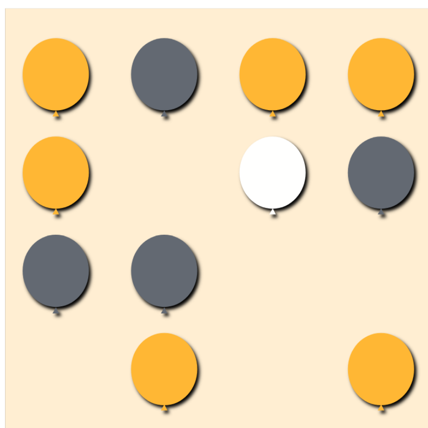
4. How much should the winner receive after paying \$1 to play so that the game is fair?

A Probability Distribution

The game attendant wants to make things more exciting! They decide to add different colored balloons and vary the prize money depending on which balloon you hit!

To account for the added expenses, the game now costs \$2 to play. If a player hits the gray balloon, they win \$4. Hitting a yellow balloon wins \$5. And if the player hits the one white balloon, they win \$10!

Let's define the random variable $X =$ net gain from a single game.



1. Find the probability distribution of X.

2. Determine $P\left(X \geq 3\right)$ and interpret the result.

3. Compute the expected value of X. Explain what this result means for the player.

4. Compute and interpret the standard deviation of the random variable X.

5. Determine a set of prize winnings after paying \$2 to play so that the game is fair.

Ring Toss

The stand next to the balloon game offers another game involving tossing rings on pegs. Let's define the random variable $Y =$ net gain from a single game of ring toss. The probability distribution of Y is shown below.

Net gain y_i	-3	2	5	11	25
Probability p_i	0.55	0.25	0.17	0.02	0.01

1. Compute the expected value and standard deviation of Y. Explain what this result means for the player.
2. Assume a guest plays both the updated Balloon Pop game and the Ring Toss game and that the outcomes of the games are independent. Let $T =$ net gain from both games. That is, let $T = X + Y$. Compute the expected value and standard deviation of T. Explain what this result means for the player.